Applications of Bispectral Analysis to Acoustic Signature Identification

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Introduction to Bispectral Analysis

The power spectrum is the frequency decomposition of signal power in the frequency domain. When this concept is extended to higher orders, the result is called a polyspectrum. More specifically, the third-order polyspectrum is referred to as the bispectrum. The bispectrum is defined as the frequency decomposition of the skewness, or third-order covariance function, of a signal.

Covariance Function:

$$c(k) = E\{x^*(t)x(t+k)\}$$

Third-order Covariance Function:

$$c_3(k,l) = E\{x^*(t)x(t+k)x(t+l)\}$$

The bispectrum provides information about signal features, such as phase coherence, that the second-order power spectrum does not. The most important use of the bispectrum lies in the detection of non-linearities. For a discrete time series, the discrete bispectrum is defined as the double Fourier transform of the third order covariance function and can be expressed in the following ways:

$$B(k,l) = \sum_{k=-\infty}^{\infty} \sum_{l=-\infty}^{\infty} c_3(k,l)\exp(-if_1 k)\exp(-if_2 l) \qquad (1)$$

$$B(k,l) = DFT^2[c_3(t_1,t_2)] = E[X(k)X(l)X^*(k+l)] \qquad (2)$$

where E denotes the expectation operator and $X(k)$ the discrete Fourier transform. The bispectrum is a function of two different frequencies f_k and f_l, and only those bifrequencies (f_k, f_l) that fall within the following domain need be computed

$\{f_k, f_l\}: 0 \le f_k \le f_s/2, \quad f_l \le f_k, \quad 2f_k + f_l \le f_s$ where f_s is the sampling frequency. This triangular region is referred to as the Principal Domain, and all bifrequencies outside of this domain are redundant due to the symmetric properties of the bispectrum. A peak in the bispectrum at the bifrequency (f_k, f_l) indicates a coupling between the three frequencies f_k, f_l and $f_m = f_k + f_l$ and their corresponding phases f_k, f_l and f_m. This

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frequency and phase coupling is the result of a quadratic type of non-linearity within the signal.

In practice, a normalization of the bispectrum is commonly used. To normalize the bispectrum, the final estimate is divided by the product of the spectral components.

$$b(k,l) = \frac{\left|\hat{B}(k,l)\right|^2}{\hat{P}(k)\hat{P}(l)\hat{P}(k+l)} \qquad (3)$$

This normalization, referred to as the bicoherence, measures the percentage of power at frequency $f_m, m = k + l$ due to wave coupling. The bicoherence removes the dependency on amplitude, or in other words, it is completely independent of the power of the three waves.

In order to better understand the bispectrum and its application, several different models, including a synthetically coupled signal (Section 1.1), a synthetic signal with additive Gaussian noise (1.2), and finally a real life seismic signal (1.4) will be examined. The synthetic signals will have a strong harmonic structure and contain various interactions among frequency components. Such frequency interactions or non-linearities will be revealed in the bispectrum.

Due to its sensitivity to non-linearities, it is believed that the bispectrum may have practical uses in the area of structural health monitoring and damage detection, as well as detecting nonlinear signals contaminated with substantial amounts of Gaussian noise. The bispectrum has been shown to be a successful indicator of fatigue cracks in cantilever beams (Rivola and White, 1998)[3]. This will be examined more thoroughly in section 1.3.

The bispectrum will then be applied to a real-life drill signal for the purpose of extracting important features from the signal (1.4). Higher order spectral analysis may provide a powerful means of signature identification and signal interpretation. Being able to distinguish an acoustic underground drilling signal such as the one studied, from some arbitrary underground signal obscured by natural "cultural noise" is of extreme importance.

1.1 A Synthetic Signal

In order to test out the properties of the bispetrum, a synthetic signal was generated in Matlab consisting of several cosine components. Specifically, sixty-four independent realizations of the signal

$$y(n) = \sum_{k=1}^{p}\sum_{i=1}^{3} a_{ki}\cos(2p\, l_{ki} n + f_{ki}) + \sum_{k=1}^{\bar{p}} \overline{a_k}\cos(2p\, \overline{l_k} n + \overline{f_k}) + g(n)$$

were generated, each one containing 64 samples. The sample rate was chosen to be 400Hz . One phase coupled triplet (p=1) was chosen at the frequencies (l_{1i}) of 40Hz, 80Hz, and 120Hz, with amplitudes (a_{1i}) of one. Notice that 40Hz and 80Hz add to 120Hz, and for this reason are said to be frequency coupled. In order to be phase coupled, the phases of the sinusoids must also have an additive relationship, with f_{11} and f_{12} summing to f_{13}. An additional harmonic for the uncoupled case (terms with an overbar) was chosen at the frequency 160Hz. This frequency of 160Hz was chosen intentionally to form a frequency coupled relationship between 40Hz and 120Hz. However, the corresponding phase of the 160Hz component was chosen randomly, and thus does not exhibit phase coupling. The bispectrum should only pick up on the terms that are frequency and phase coupled. In this initial case, g(n), representing additive colored Gaussian noise, was chosen to be zero. The effect of additive noise on the Bispectrum will be explored in the next section.

In Figure 1 it can be seen that both the bispectrum and the bicoherence reveal the phase and frequency relationship between 40Hz and 80Hz . The bispectrum does not show a peak at 40Hz and 120Hz due to the lack of phase relation between these two frequency components with the sum component of 160Hz.

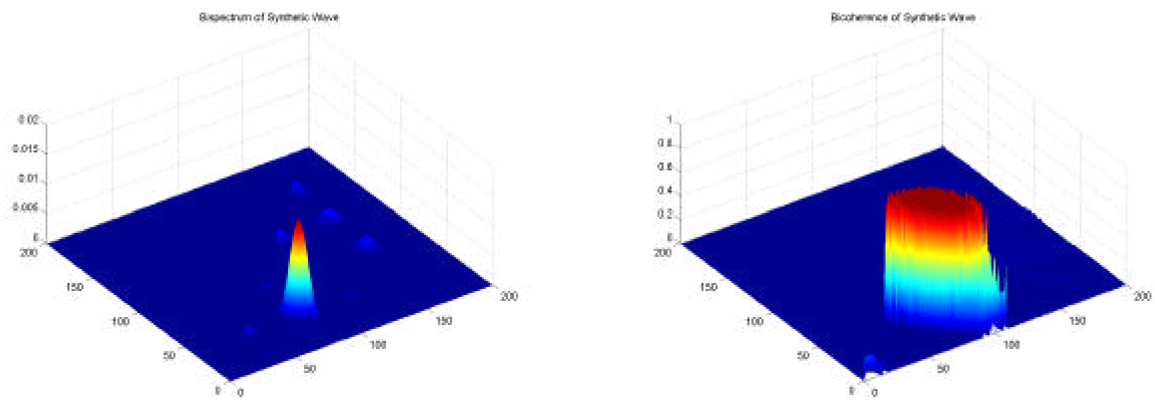


Figure 1: (a) Bispectrum of synthetic signal, max peak at (40Hz,80Hz) as expected. (b) Bicoherence of synthetic signal, broad peak centered at (40Hz,80Hz), magnitude of one.

1.2 A Synthetic With Noise

Gaussian distributed processes are completely characterized by their mean and variance or first and second moments. The bispectrum is the frequency decomposition of the third order cumulant. The third order cumulant of such a Gaussian process will yield a zero bispectrum across all frequencies. For this reason, higher order measures such as the bispectrum are insensitive to Gaussian noise. A harmonic signal x(t) in Gaussian noise should have the same bispectrum as the harmonic signal x(t) minus the additive noise. This ability to suppress noise is one benefit of the bispectrum over the traditional power spectrum. As an example let us again consider the following equation:

$y(n) = \sum_{k=1}^{p}\sum_{i=1}^{3} a_{ki}\cos(2p\,l_{ki}n + f_{ki}) + \sum_{k=1}^{\bar{p}} \overline{a_k}\cos(2p\,\overline{l_k}n + \overline{f_k}) + g(n)$, this time where $g(n)$ represents additive Gaussian noise with a variance of 1.5.

Based on the above discussion, no significant difference should exist between the bispectrum of this signal and the noise-free one of section 1.1. The power spectrum of the noise-free synthetic reveals peaks at all generated harmonics, 40,80,120, and 160 Hz. The addition of colored Gaussian noise to the synthetic clearly obscures the underlying signal of interest. The power spectrum now contains several additional peaks which could lead to a misinterpretation of the signal. Figure 2 shows the difference in the power spectra with the addition of colored Gaussian noise.

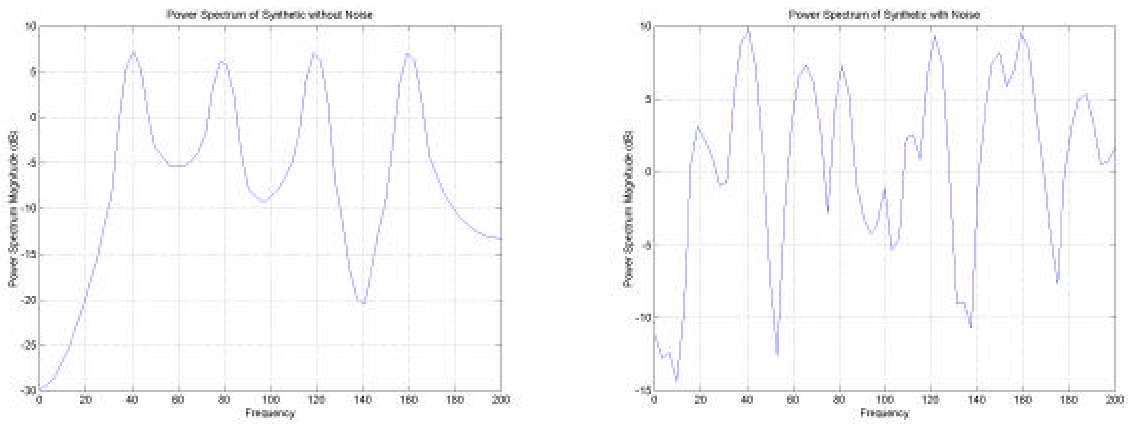


Figure 2: (a) Power spectrum of synthetic signal. (b) Power spectrum of synthetic signal plus noise. Addition of noise obscures original signal.

The bispectrum that appears in Figure 3 verifies that it is blind to the noise.

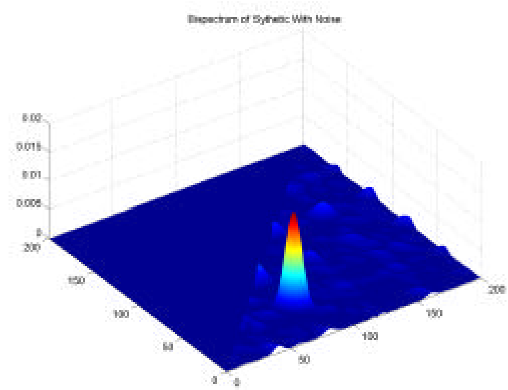


Figure 3: Bispectrum of Synthetic with noise, completely insensitive to noise. Peak at (40Hz,80Hz). Bicoherence displays same results.

1.3 Practical Application to Damage Detection

Using simulations as well as experimental data, Rivola and White [3], study the applicability of bispectral analysis to the detection of fatigue cracks in a beam. They also compare the results of bispectral analysis to first order spectral measures.

In their simulation, they develop a simple single-degree-of-freedom model of a cracked beam with two states — crack open and crack closed (depending on which way the beam is flexed). With the crack closed, the beam acts as a homogeneous beam with no crack. When the crack is open, a local reduction of flexural rigidity occurs at the point of the crack. Numerical studies of this model show changes in the magnitude of superharmonic components with fracturing; however, the magnitudes of the changes are very small, especially when the stiffness ratio is close to one. From these results, Rivola and White conclude "[I]n the case of a beam, the spectrum of the experimental response could not be able to detect a crack at the early stages of development, because the harmonic components might be obscured, for example, by the noise." They are led, therefore, to consider more sophisticated processing techniques; in particular, the bispectrum.

In order to compare results across frequencies, they normalize the bispectrum into a form known as the bicoherence, which equalizes the variance at different frequencies and ranges the values from zero to one. They find that when the system is linear (stiffness ratio = 1), the bicoherence is approximately flat, with the maximum value about 0.06. They note that "On the other hand, as soon as the stiffness ratio decreases, this is to say, the system becomes nonlinear, the bicoherence shows drastic deviations." When the stiffness ratio is 0.95, the bicoherence rises to 0.67. When the stiffness ratio is 0.90, the bicoherence rises to 0.84. They conclude that "Therefore, the bicoherence apears to be very sensitive to the presence of nonlinearities" [p.902].

They then follow up their simulation with an experiment on a straight bronze beam, 450 mm. in length, 10 x 10 mm. in cross-section. They mounted the beam in a three point bending fixture and excited it with white Gaussian noise. They computed the bicoherence from the measured acceleration response under three conditions: beam undamaged, and beam with two degrees of transverse cracking. They find that the effect of the cracking "barely apears in the power spectrum, but it is clearly evident in the bicoherence."

Overall, Rivola and White conclude that the bicoherence is a sensitive way to detect nonlinearities, and that it is likely to be useful in diagnosing structural damage. We agree with this conclusion, but we note certain caveats in the in the application of bispectral analyses to real-world situations. First, both the simulations and the experimental data are rather simplistic when compared with bridges and buildings and other structures people actually use, which are likely to exhibit nonlinear characteristics even when undamaged. Second, the extent of the nonlinearities may vary considerably with environmental conditions such as ice, rain, heating and cooling, wind loads, etc. The next logical step seems to be to apply the bispectrum to simulated data of known failure modes on larger structures of interest. If that is found to be successful, then bispectral analysis may prove to be useful in full-scale structural damage detection.

1.4 Investigating a Real-World Signal – The Tunnel Boring Machine

Having examined the various simulated cases above and the previously researched experiment, it is important to now determine if bispectral analysis can indeed be applied to a real world signal and provide interesting results. In order to address this question, we now apply the bispectrum to a seismic signal of an episode of underground drilling activity. Can higher order spectral analysis be used to characterize the signal and can nonlinearities be detected within the signal? This section seeks to answer those questions.

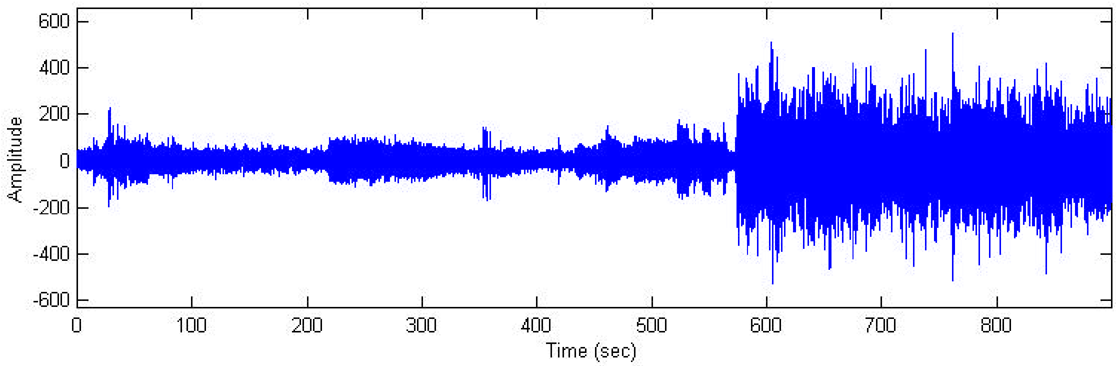


Figure 4: Time series of 900 seconds of underground activity recorded by a seismometer and sampled at 500Hz. The actual drilling starts around 580 seconds.

Figure 4 shows the time history of 900 seconds of underground activity as recorded by a seismometer. The signal was sampled at 500Hz. The increase in amplitude around 580 seconds is indicative of where the tunnel boring activity begins. The initial step in understanding what frequency components and corresponding activities where contained in the signal was to examine the spectrogram. The spectrogram, seen in Figure 5, shows several frequency peaks. The tunnel boring machine extends from 5Hz to 55Hz. The peaks ranging from 90Hz to 250Hz which occur every 300 seconds are due to the data recording system. There are several other as of yet unexplained peaks visible in the spectrogram (ex: 19Hz, 55Hz).

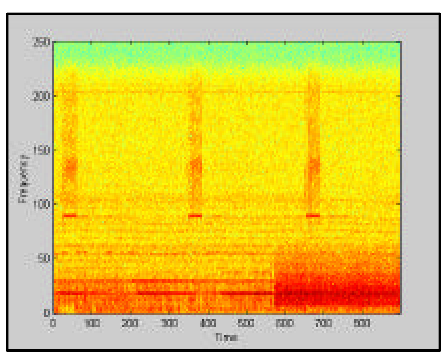
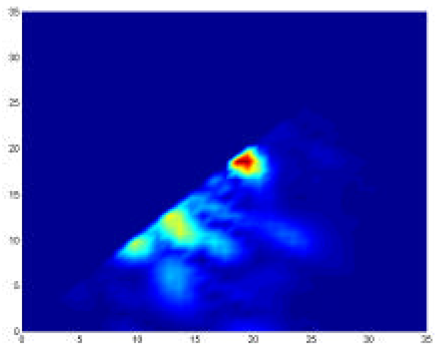


Figure 4: Spectrogram of tunnel boring machine (TBM).

The data was processed using both the bispectrum and the bicoherence. The data set was split up in order to better analyze it. A 100 second section of known tunnel boring activity was processed and for comparison, a 100 second section of fairly low amplitude, stable activity was also processed. The idea behind comparing these two sections was to distinguish features specific to the drill from the other activity in the signal. The bispectrum of the drill section indicated a coupling of 19Hz with itself creating the additive frequency at approximately 40Hz. The bicoherence also showed a peak at (19Hz, 19Hz). However, it showed an additional peak at (4Hz, 4Hz). This peak did not show up in the bispectrum perhaps because of the small power at 4Hz. The power spectra of the two sections both showed high power at 19Hz, but for the drill section power spectrum, the power at 19Hz was twice that of the non-drill. One conclusion that can be drawn from this is that the drill has a resonant frequency at 19Hz. The harmonic at 19Hz was originally attributed solely to the Ventilation system, but this analysis leads to the idea that 19Hz is also part of the drilling aparatus, as is 4Hz. Figure 5 shows the bispectrum and bicoherence plots of the 100 second drilling section.



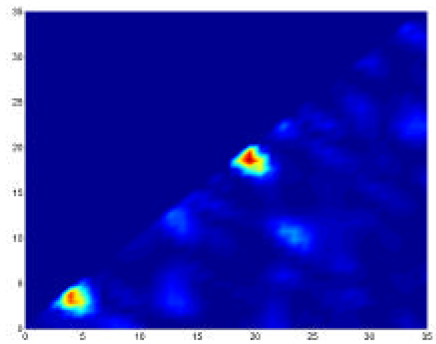


Figure 5: (a) Contour plot of the bispectrum (x and y are frequency axes) of 100 second long TBM activity. Maximum peak at (19Hz,19Hz) indicating wave coupling among 19Hz, 19Hz, and 40Hz. (b) Bicoherence of same TBM activity with max peaks at (19Hz, 19hz) and (4Hz, 4Hz).

In conclusion, it is important to state that more research needs to be done on this signal and other similar signals of interest. This research has proved that the bispectrum provides information that the traditional power spectrum does not, and is useful in identifying important signal signatures.

Some factors that should be considered in future research include the following:

- Should the time series be subsampled to focus more attention on the low frequency components of the drill signal?
- What are the optimum parameters for bispectral analysis?
- How do the higher order statistical measures correlate with second order spectral measures?
- Will the fourth order polyspectra provide more information of interest?

1.5 Future Work and Expected Results

Bispectral Analysis may have further applications in the area of Structural Health Monitoring. A Damage Detection Study was conducted on two concrete columns that were incrementally loaded to failure. The data from this experiment will be processed using the bispectrum and the bicoherence. It is suspected that a rise in bicoherence values of the acceleration response will accompany the introduction of damage (fatigue cracks) into the column. Faults or cracks are accompanied by a change in the vibrational signature of the structure, most often these changes are in the form of non-linearities. The bicoherence is designed to extract such information.

References

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Acknowledgements

George Papcun, Dave Alexander, Murray Wolinsky